

Fast scrambling as Brownian motion in a fluid with negative viscosity

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Abstract

It is shown that the fast scrambling of information in a black hole can be viewed as Brownian motion of information in a fluid with negative viscosity (and negative temperature). It is argued that a non-local character of the fast scrambling is only an illusion; the stretched horizon with negative viscosity is an amplifying medium that mimics non-locality and superluminal communication.

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There are two fundamental problems in black hole physics:

1. What are the degrees of freedom (d.o.f.) responsible for the Bekenstein-Hawking black hole entropy?
2. How does they process quantum information?

Usually these problems are discussed separately in the literature. But they are closely related; knowing the d.o.f. we can answer the second question and vice versa. The Bekenstein-Hawking entropy is a concept defined in the rest frame of an external fiducial (fixed r) observer. From the viewpoint of the observer, the black hole's d.o.f. reside on the stretched horizon, a two-dimensional timelike surface ('membrane') located roughly one Planck length away from the event horizon and endowed with certain mechanical, electrical and thermal properties. The number of the d.o.f. is of order the area of the horizon in Planck units. These d.o.f. absorb, thermalize and re-emits all information in the form of Hawking radiation. In particular, from the point of view of the fiducial observer, a charge falling into the horizon spreads over the entire horizon. This is in contrast with the point of view of a free infalling observer who does not experience the horizon. The principle of the black hole complementarity states, however, that there is no way to compare observations inside and outside the horizon to find a violation of the quantum no-cloning theorem.

From the viewpoint of the fiducial observer, the black hole's d.o.f. thermalize quantum information in the same way as the charge. The thermalization or scrambling time t_{scr} is one of the most important parameters of the process [1], [2]. For a system with N d.o.f., it is a measure of how long it takes for information about a small $O(1)$ perturbation in a system to spread over the $O(N)$ system's d.o.f. The crucial importance of the scrambling time consists in the fact that knowing it we can get insights into the nature of the black hole's d.o.f. responsible for the Bekenstein-Hawking entropy and into the mechanism of information retrieval from evaporating black holes. Can this time be made arbitrary small?

In [3], Hayden and Preskill found that for a black hole with temperature T and entropy S

$$t_{\text{scr}}T \sim \log S, \quad (1)$$

and argued that we must put a lower bound on the scrambling time to avoid a violation of the quantum no-cloning theorem. Therefore, $t_{\text{scr}}T \geq \log S$. However, this is a much shorter time scale as a function of the entropy than the time that a Brownian particle would take to diffuse over the entire horizon. Indeed, by using diffusion approximation, Seiko and Susskind [1], and also Susskind [2], showed that for ordinary local systems, such as quantum field theories or fluids,

$$t_{\text{scr}}T \sim S^{\frac{2}{d}}, \quad (2)$$

where d is the number of spatial dimensions. Comparing (1) and (2), Seiko and Susskind conjectured that black holes are the fastest scramblers in the nature. The stretched horizon is a two-dimensional system. But if measured by its scrambling time it is more like infinite dimensional systems. Thus, Seiko and Susskind concluded that the fast scrambling is a result of the non-local interactions between the black hole's d.o.f.; local interactions, in their opinion, would lead to slow diffusion as in Brownian motion.

In this essay we shall show that this conclusion is not quite true. The point is that it is valid only for fluids with positive viscosity. However, as Damour showed [4], the stretched horizon

behaves as a fluid with a *negative viscosity*. As a result, a Brownian particle moving in such a fluid will not slow down, but accelerate; as will be demonstrated below, the mean-square displacement of the particle grows exponentially with time. By using this fact, we shall show that the fast scrambling in black holes can be interpreted as Brownian motion of information in the stretched horizon with negative viscosity and negative temperature. For alternative interpretations, see [5]-[8].

At first, we shall illustrate our idea on a simple example and then consider a more realistic model. We start with the Langevin equation for a Brownian particle of mass m immersed in a fluid with temperature T and viscosity ζ

$$m\ddot{\mathbf{x}} = -\alpha\dot{\mathbf{x}} + \mathbf{f}(t), \quad (3)$$

where α is the frictional constant proportional to viscosity (for a spherical particle with radius r , $\alpha = 6\pi\zeta r$) and $\mathbf{f}(t)$ is a random force with zero average, $\langle \mathbf{f}(t) \rangle = 0$. It can be shown (see any course on stochastic processes) that the general expression for the mean-square displacement of the particle in two spatial dimensions is given by

$$\langle x^2 \rangle = \frac{4k_B T}{\alpha} [t - \gamma^{-1}(1 - e^{-\gamma t})], \quad (4)$$

where $\gamma = \alpha/m$. For $\zeta > 0$ and $t \gg \gamma^{-1}$ we get the famous result $\langle x^2 \rangle = 4Dt$, where D is the diffusion coefficient, $D = k_B T/\alpha$. According to kinetic theory $D \sim l_0 \bar{p}$, where l_0 is the mean free path and \bar{p} is the average momentum of a molecule in thermal equilibrium. Then, by using the fact that $\langle x^2 \rangle \sim N l_0^2$ and $\bar{p}^2 \sim T$, we get $t_D T \sim N l_0 \bar{p}$, where N is the number of molecules and t_D is the time of diffusion. Following Seiko and Susskind, we assume that for ordinary local systems the scrambling time is at least as long as the diffusion time. When the de Broglie wavelength becomes of order l_0 , we enter into the regime of strongly correlated quantum fluids and we have $t_{\text{scr}} T \sim N$. If the total entropy of the fluid is of order the number of molecules, we may also write $t_{\text{scr}} T \sim S$. This is the old result of Seiko and Susskind in two dimensions.

Consider now the case $\zeta < 0$ and $t \gg |\gamma^{-1}|$. In this case we get

$$\langle x^2 \rangle = 4D\gamma^{-1}e^{\gamma t}, \quad (5)$$

a completely new result. Since $|\gamma^{-1}| \sim l_0 \sim T^{-1}$, we get $t_{\text{scr}} T \sim \log S$ in the regime of strongly correlated quantum fluids. This is of the same form as (1). Therefore, Brownian motion in a fluid with negative viscosity can be as fast as the scrambling in black hole. The reason for this is that, due to internal instability, such a fluid behaves as an amplifying medium. Amplification of a macroscopic degree of freedom, such as a laser or maser field, or the motion of a Brownian particle, is a phenomenon of enormous scientific and technological importance. In classical thermodynamics, amplification is not possible in thermal equilibrium at positive temperature. In contrast, due to the fluctuation-dissipation theorems, amplification is a natural phenomenon, and fully consistent with the second law of thermodynamics, in a heat bath at *negative temperature*. Indeed, according to the fluctuation-dissipation theorem in the Nyquist form [9], the spectral density of the correlation function for the random force

$\mathbf{f}(t)$ is, up to a numerical factor, the product of the friction coefficient γ and the temperature T . It is always *positive*. Therefore, if the viscosity ζ is negative, then so is the temperature T . Thus, the exponential growth of the mean free path with time in (5) can be viewed as a result of Brownian motion in a fluid with negative viscosity and negative temperature. A difficulty arises, however, when we consider black holes. The Hawking temperature is positive. Where does the negative temperature come from?

We start answering the question with an important note. The Schwarzschild metric is featureless: the black holes have no hairs. What happens if we get closer to the event horizon? In the near-horizon approximation, the Euclidean metric of a black hole takes the Rindler form

$$ds_E \approx \rho^2 d(g_H t_E)^2 + d\rho^2 + \frac{1}{4g_H^2} d\Omega^2, \quad (6)$$

where ρ is the proper distance from the horizon and g_H is the surface gravity, $g_H = 1/4GM$. Since t_E appears in the metric only through its square, the classical solutions will be the same metrics for positive and negative temperature $T_H = t_E^{-1}$. In [10], Braden *et al.* considered the question of determining the density of states of the microcanonical ensemble for a black hole in a thermally isolated box containing a fixed amount of energy. They showed (see also Louko and Whiting [11]), that negative temperature arises in a natural and consistent way for a singled-valued action of black-hole thermodynamics. However, their treatment does not indicate how a state of gravitational field at negative temperature should be prepared. We shall not look for the negative temperature states of gravitational field in this essay. Instead, we shall investigate one detailed example of the stretched-horizon dynamics, which demonstrates explicitly the appearance of negative temperatures in black holes. The argument goes as follows. According to the membrane paradigm [12], the dynamical equations of the stretched horizon are equivalent to the stretched-horizon fluid equations. In particular, for a Schwarzschild black hole, the focusing (Raychaudhuri) equation for the horizon expansion θ_H is identical to the energy conservation for a fluid with surface energy density $\Sigma_H = -\theta_H/8\pi G$, surface pressure $P_H = g_H/8\pi G$, shear and bulk viscosity $\eta_H = 1/16\pi G$ and $\zeta_H = -1/16\pi G$, respectively. For a complete understanding of the horizon-fluid dynamics we must also take into account the thermodynamical properties of a black hole. In particular, we shall regard a small patch of the horizon-fluid of area ΔA as endowed with a temperature $T_H = g_H/2\pi$ and entropy $\Delta S_H = \Delta A/4G$. Then the energy conservation for the patch, when rewritten using the expression for Σ_H , P_H , η_H , ζ_H , and $\theta_H = (1/\Delta A)(d\Delta A/dt)$, becomes the heating equation (without shear terms) [13], [14]

$$\frac{d^2 \Delta S_H}{dt^2} = 2\pi T_H \frac{d\Delta S_H}{dt} - 2\pi \Delta A (\zeta_H \theta_H^2 + \mathcal{F}_H), \quad (7)$$

where \mathcal{F}_H is the flux of entropy across the horizon from the external universe. For small θ_H , the equation reduces to

$$\frac{d^2 \Delta S_H}{dt^2} = 2\pi T_H \frac{d\Delta S_H}{dt} - 2\pi \Delta A \mathcal{F}_H. \quad (8)$$

In this form, it is similar to the Langevin equation (3) with *negative* friction coefficient $-2\pi T_H$ and random force $2\pi \Delta A \mathcal{F}_H$. Indeed, Bhattacharya and Shankaranarayanan showed in [15] and

[16] that Raychaudhuri equation for the horizon expansion θ_H can be viewed as a Langevin equation. They noticed that ΔS_H ($\sqrt{\Delta A}$, in their work) increases exponentially with time $\Delta S_H \sim e^{t T_H}$. However, they did not realize that this increase is related to the *negative* temperature T_H . In our opinion, it is the negative temperature T_H that causes the exponential increase of ΔS_H . Therefore, we conclude that propagation of a small perturbation in the horizon-fluid can be regarded as Brownian motion in a fluid at the negative temperature T_H and the time required for the perturbation to diffuse in the stretched-horizon fluid is the same as the scrambling time (1).

It must be realized, however, that for all real system the states with negative temperatures are, strictly speaking, not equilibrium states but merely metastable states. As a consequence of this, we can speak of negative temperature only in some conditional sense. The concept of negative temperature has a still more conditional sense in the theory of quantum amplifiers of electromagnetic waves, lasers and masers. The point is that the laser (maser) medium is not in a thermal state but in a steady state. Therefore, it is not in thermal equilibrium and cannot have a temperature. The concept of negative temperature is used in lasers (masers) only as a conditional quantity characterizing the inversion population condition. In this essay we consider the negative temperature of the stretched horizon-fluid in the same sense.

In our opinion, this is a key to understanding of the mystery of non-locality in the fast scrambling. The point is that the stretched horizon with negative temperature (and viscosity) behaves as an inverted or amplifying medium. As is well known [17]-[19], in such a medium, the occurrence of a perturbation at a certain point is associated with the amplification of a part of harmonics already available at this point due to the system's instability rather than with the energy (and information) transmission. For example, a Gaussian wave packet has wings that extend from $-\infty$ to $+\infty$, so that the wave packet is literally everywhere at all times. It turns out [18], the inverted medium temporarily borrows a part of its stored energy to the forward tail of the wave packet in reshaping process that moves the peak of the wave packet forward in time. Note that, in contrast to the wave equation, the diffusion equation being only first order in time, implies that perturbation must propagate with infinite speed. However, at the fundamental level, there is no superluminal communication. Moreover, we can even find a situation [19], in which the output wave packet leaves the inverted medium before the peak of the input wave packet enters. This is very much like the teleological behavior of the black hole's horizon. However, in this case, there is no violation of causality. Since the stretched-horizon fluid behaves as an inverted or amplifying medium, we conclude that there is no non-locality in the fast scrambling process. The stretched horizon mimics non-locality; we have only an illusion.

It is possible that instability caused by the negative temperature (and negative viscosity) is related with dynamical chaos. In particular, it is believed [20], [21] that the rate of scrambling can be interpreted in terms of chaos as a Lyapunov exponent λ_L , $t_{\text{scr}} \sim \lambda_L^{-1} \log S$. In this connection it may be noted that the Lyapunov exponent for the spreading process was first introduced and determined in [22].

The existence of a lower bound on the scrambling time seems to imply that the relation (1) has its origin in the uncertainty relation. Therefore, we want to conclude this essay by demonstrating how (1) can be derived from the uncertainty relation. As we know from

quantum mechanics [23], the relation $\Delta E \Delta x \sim \dot{x}$ exists between the quantum uncertainties of energy and of some quantity x , \dot{x} being the classical rate of change of x . Let us suppose that quantum uncertainty of x is of the same order as x , so that $\Delta x \sim x$. Then we have $\Delta E x \sim \dot{x}$. It follows from this that

$$t \Delta E \sim \log x. \quad (9)$$

Let us relate all these quantities with a black hole. As is well known, if a fiducial observer who is a distance of order the Schwarzschild radius R_S from the horizon drops a freely falling quantum information into the black hole, it takes Schwarzschild time of order $t \sim R_S \log R_S$ for the information to reach the stretched horizon, and conversely it takes Schwarzschild time of order $R_S \log R_S$ for quanta emitted from the stretch horizon to reach the observers detector. Indeed,

$$t \sim \int_{2R_S}^{R_S+\delta} \frac{dr}{\sqrt{1 - \frac{R_S}{r}}} \simeq R_S \log R_S, \quad (10)$$

where (restoring for a moment the Plank constant) $\delta \sim l_P^2/R_S$; remember that the stretched horizon is located about one Planck unit of proper distance above the event horizon. Let us now identify x with R_S . Since we have $2R_S$ in the lower limit of (10), we can put $\Delta R_S \sim R_S$. For a black hole ΔE is of order the black hole's temperature, $\Delta E \sim T$. Finally, we get the scrambling time (1) derived from the uncertainty relation.

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